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The phase velocity of acoustic perturbations near a vibration excitation source☆

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ABSTRACT

The plane problem of the forced vibrations of an elastic half-space due to the action of a time-periodic force, acting along the tangent to the surface of a medium is considered. It is shown that in the near zone the phase velocity of surface acoustic perturbations varies considerably with distance from the vibration source and considerably exceeds the Rayleigh wave velocity.

The problem considered here arises when investigating the acoustic properties of soft biological tissues.[1–6] Experimental data closest to it have been given by Voronkov[1] and by Shorokhov and his co-authors.[6]

Consider, in a Cartesian system of coordinates x, y, z, the problem of the motion of points of the surface (x, y) of an elastic half-space $z \geq 0$ due to the action of a tangential, time-periodic force, applied to part of the surface $|x| \leq a$, directed along the x axis and constant along the y axis. The normal and tangential components of the stress tensor

$$z = 0\colon \sigma_{zz}(x) = 0, \ \sigma_{zx}(x) = 2\mu f(x)\exp(-i\omega t); \ \ f(x) \equiv 0 \ \text{ for } \ |x| > a \tag{1}$$

are specified at $z = 0$. Here μ is the shear modulus of the elastic medium, ω is the angular frequency of the external force and $f(x)$ is its absolute value, divided by 2μ.

In experiments[6] the vibrations of the surface of an elastic medium were excited by a thin wide probe (everywhere henceforth we have only these experiments in mind). To model this situation, we will assume that the shear force acting on the half-space, is concentrated along the straight line $x = 0$, in which case $f(x) = F_0\delta(x)$, where $F_0 = \text{const}$ and $\delta(x)$ is the delta function.

With this form of vibration excitation in the half-space, both longitudinal and shear waves occur. The formal solution of this problem can be written in the form of integrals over an infinite straight line using a Fourier transformation with respect to x (see, for example, Ref. 7), where the integrand has singularities, oscillates and decreases slowly at infinity. In order to ascribe meaning to these integrals and to obtain formulae suitable for calculations, the contour of integration in the complex plane was transformed in the same way as was done previously in Ref. 8 for a similar problem.

The formula obtained for the horizontal displacements, used for the calculations, has the form

$$u_x(x, 0) = \frac{2k_2^2F_0}{\pi}\exp(-i\omega t)\left\{ i\pi\frac{\sqrt{k_R^2 - k_2^2}}{N'(k_R)}\exp(ik_Rx) - \int_0^\infty \frac{s_2\exp(-\eta x)d\eta}{(\eta^2 + s_2^2)^2 - 4\eta^2 s_1 s_2} - \right.$$
$$\left. - \int_0^{k_1} \frac{ig_2\exp(i\xi x)d\xi}{(\xi^2 - g_2^2)^2 + 4\xi^2 g_1 g_2} - \int_{k_1}^{k_2} \frac{ig_2(\xi^2 - g_2^2)^2\exp(i\xi x)d\xi}{(\xi^2 - g_2^2)^4 + 16\xi^4\gamma_1^2 g_2^2} \right\} \tag{2}$$

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[1] Voronkov, V.N. An investigation of the mechanical properties of human skin under normal conditions and in pathological states: Candidate Dissertation, Institute of Cell Biophysics of the Russian Academy of Sciences, 1993.

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Here

$$N(\xi) = (\xi^2+\gamma_2^2)^2 - 4\xi^2\gamma_1\gamma_2, \quad k_1 = \omega\sqrt{\rho/(\lambda+2\mu)}, \quad k_2 = \omega\sqrt{\rho/\mu}$$

$$\gamma_j = \sqrt{\xi^2 - k_j^2}, \quad s_j = \sqrt{\eta^2 + k_j^2}, \quad g_j = \sqrt{k_j^2 - \xi^2}; \quad j = 1, 2$$

k_R is the wave number of the Rayleigh wave, λ and μ are Lamé constants and ρ is the density of the medium.

In experiments[6] the phase velocity of propagation of the perturbations was found from the formula

$$\upsilon_L = \frac{\omega}{k_L}; \quad k_L = \frac{\Delta\Phi}{L} = \frac{1}{L}\int_0^L k(x)dx, \quad k(x) = \frac{d\Phi}{dx}, \quad \Phi = \operatorname{arctg}\frac{\operatorname{Im}(u_x)}{\operatorname{Re}(u_x)} \tag{3}$$

where k_L is the average wave number, L is the distance from the source (at the point $x=0$) to the point of measurement, and $\Delta\Phi$ is the phase difference of the steady vibrations at these points.

Obviously, in the near zone, the perturbations do not have the form of a plane wave $\exp(ikx - i\omega t)$. However, if we use the "local" wave number $k(x)$, we can determine the local phase velocity of the perturbation in question from the formula

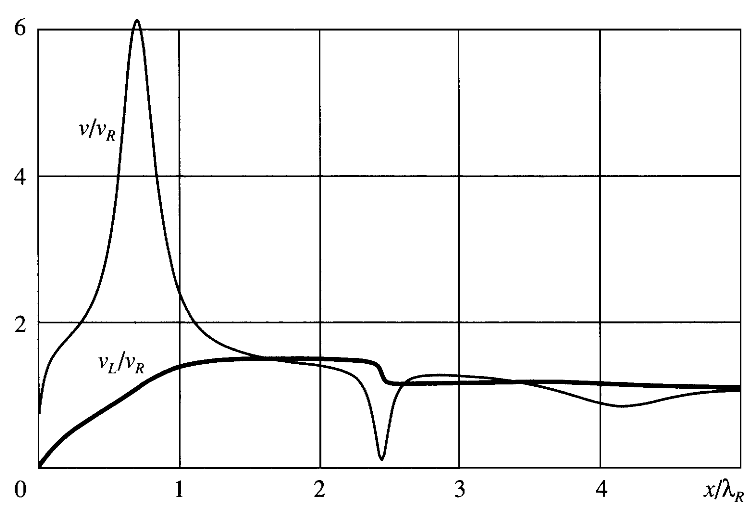
$$\upsilon(x) = \omega/k(x) \tag{4}$$

It follows from formulae (2)–(4) that the local phase velocity v is proportional to the Rayleigh wave velocity $v_R = \omega/k_R$ with a coefficient which depends only on the distance L from the vibration excitation source and Poisson's ratio. This also holds for the mean phase velocity v_L.

Using formulae (2)–(4) we calculated the local and mean phase velocity as a function of the dimensionless distance x/λ_R from the vibration excitation source (λ_R is the Rayleigh wavelength). Since the experiments described in Ref. 6 were carried out on models made of soft rubber, the calculations were carried out in the range of Poisson's ratios from 0.3 to 0.49. It turned out that, in this range, the value of Poisson's ratio depends slightly on the dependence of v/v_R and v_L/v_R on c; they are shown in the figure for a value of Poisson's ratio of 0.49.

It can be seen that the local phase velocity close to the source changes considerably with distance. At a distance of x_{max} equal to about ¾ of the Rayleigh wavelength λ_R, there is a maximum (the phase velocity is approximately six times greater than the Rayleigh wave velocity), while at a distance of x_{min} equal to $2.5\lambda_R$, there is a minimum, (the phase velocity falls almost to zero). For example, for a frequency of 5 kHz and a shear modulus of 10^6 Pa, x_{max} = 4.5 mm and x_{min} = 15 mm. The mean phase velocity varies more smoothly.

The ratio of the mean phase velocity to the Rayleigh wave velocity v_L/v_R calculated from formula (3) is represented by the thick curve. The mean phase velocity when $x_{max} < x < x_{min}$ corresponds to the results of experiments.[6]



The results obtained show why, near the perturbation source, where the Rayleigh wave has already been formed and the phase velocity is not constant, the mean phase velocity measured experimentally[6] varies only slightly (in the range of ±10% of the mean value). Moreover, we have shown that there is a considerable difference between the experimentally measured velocity in this region and the Rayleigh wave velocity and we have given the dependence of the limits of this "region of constancy" on the Rayleigh wavelength, i.e. on the elastic properties of the medium and the perturbation frequency. It can be assumed that the value of the universal profile of the dependence of the velocity on the distance obtained (which depends slightly only on Poisson's ratio) is outside the framework of the explanation of the results of experiments presented in Ref. 6 and is of more general interest.

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